INSTABILITIES IN FREE-SURFACE HARTMANN FLOW AT LOW MAGNETIC PRANDTL NUMBERS

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INTRODUCTION

Free-surface Hartmann flow is the parallel flow of a viscous electrically conducting capillary fluid on a planar surface, subject to gravity and a flow-normal magnetic field. Flows of this type arise in a variety of industrial and astrophysical contexts, including liquid-metal blankets in fusion devices, and plasma-ocean models of neutron stars and white dwarfs. Typically, the Reynolds number $\mathrm{Re}>10^4$ is high, and the background magnetic field is strong (Ha>100, where Ha is the Hartmann number). On the other hand, the magnetic Prandtl number Pm of laboratory fluids is small (for liquid metals, $\mathrm{Pm}<10^{-5}$), as is the case in a number of astrophysical models.

In the absence of MHD effects, free-surface flow exhibits the so-called *soft* and *hard instability modes* (1), where the former is a surface wave destabilized by viscous stresses acting on the free surface, whereas the latter is a shear mode destabilized by positive Reynolds stress associated with a critical layer. Our main objective is to study the influence of the external magnetic field on these two instabilities, working in the regime $\mathrm{Pm}<10^{-4}$, We also consider the so-called *inductionless limit*, $\mathrm{Pm}\to 0$, where magnetic-field perturbations diffuse infinitely fast, and the only MHD effects are Lorentz forces arising from currents induced by the perturbed fluid motion within the magnetic field.

When Pm is small the magnetic field generally stabilizes the flow. In particular, in *channel Hartmann flow* (i.e. plane Poiseuille flow modified by a flow-normal magnetic field) with insulating walls, the critical Reynolds number Re_c for instability increases monotonically with Ha, and for $\mathrm{Pm}\leq 10^{-4}$ it exhibits a mild, $O(10^{-4})$, decrease compared to its value in the inductionless limit (2). In free-surface flow, however we find that while the critical Reynolds number of the hard mode is equally insensitive to $\mathrm{Pm}\ll 1$ as in channel flow, the sof mode's behavior differs markedly between the small-Pm and inductionless cases (3).

NUMERICAL METHOD

We have developed a spectral Galerkin method to solve the coupled Orr-Sommerfeld and induction equations (5), which, in conjunction with suitable stress conditions at the free surface and continuity conditions for the magnetic field, govern the linear stability of free-surface MHD. Our scheme's discrete bases for the velocity and magnetic fields consist of linear combinations of Legendre polynomials, chosen according to the order of the Sobolev spaces of the continuous problem. The orthogonality properties of the bases solve the matrix-coefficient growth problem and eigenvalue-eigenfunction pairs can be computed stably at spectral orders at least as large as p = 3,000 with p-independent roundoff error.

RESULTS

Because it is a critical-layer instability (moderately modified by the presence of the free surface), the hard mode is found to exhibit similar behavior to the even unstable mode in channel flow, in terms of both of the weak influence of Pm on its neutral-stability curve, and the dependence of its critical Reynolds number Re_c on Ha. In contrast, the stability properties of the soft mode differ markedly between problems with small, but nonzero, Pm, and their counterparts in the inductionless limit. Notably, the soft mode's critical Reynolds number grows exponentially with Ha in inductionless problems, but when Pm is nonzero that growth is suppressed to either a sublinearly increasing, or decreasing function of Ha (respectively for insulating and conducting-wall problems). In the insulating-wall case, we also observe pairs of counter-propagating Alfvén waves, the upstream-propagating wave undergoing an instability at high Alfvén numbers.

We attribute the observed Pm-sensitivity of the soft instability to the strong-field behavior of the participating inductionless mode, which, even though stabilized by the magnetic field, approaches neutral stability as Ha grows. This near-equilibrium is consistent with a balance between Lorentz and gravitational forces, and renders the mode susceptible to effects associated with nonzero magnetic-field perturbations, even when the magnetic diffusivity is large. In particular, magnetic field perturbations influence the soft instability via (i) the Lorentz force arising from the steady-state induced current, and (ii) the presence of a magnetic modes in the spectrum, which is coupled to the soft mode by the basic flow, altering its stability properties. The boundary conditions play a major role in both of these effects, since they determine (i) the properties of the steady-state magnetic field, and (ii) the presence or not of the magnetic mode (that mode is not part of the spectrum of conducting-wall problems). In general, our analysis indicates that the inductionless approximation must be used with caution when dealing with free-surface MHD.

REFERENCES

1. Yih, C. S. 1963 *Phys. Fluids* **6** (3), 321; De Bruin, G. J. 1974 J. Eng. Math. **8** (3), 259
2. Takashima, M. 1996 *Fluid Dyn. Res.* **17**, 293
3. Giannakis, D., Rosner, R. & Fischer, P. 2008, *J. Fluid Mech.* (submitted)
4. Giannakis, D., Fischer, P. & Rosner, R. 2008, *J. Comput. Phys.* (submitted)

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